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Initial fields and instability in the classical model of the heavy-ion collision

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Color Glass Condensate (CGC) provides a classical description of dense gluon matter at high energies. Using the McLerran-Venugopalan (MV) model we calculate the initial energy density $\varepsilon(\tau)$ in the early stage of the relativistic nucleus-nucleus collision. Our analytical formula reproduces the quantitative results from lattice discretized simulations and leads to an estimate $\varepsilon(\tau = 0.1\text{ fm}) = 40 \sim 50\ \text{GeV}\cdot\text{fm}^{-3}$ in the Au-Au collision at RHIC energy. We then formulate instability with respect to soft fluctuations that violate boost invariance inherent in hard CGC backgrounds. We find unstable modes arising, which is attributed to ensemble average over the initial CGC fields.

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In the relativistic heavy-ion collision Color Glass Condensate (CGC) describes the initial state of energetic gluon matter with the transverse momentum p_t up to the saturation scale Q_s which universally characterizes the hadron or nucleus wavefunction in the small-x regime [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Given the scale Q_s at a certain value of Bjorken's x, the gluon distribution probed by processes with $Q^2 \ll Q_s^2$ is so dense that coherent fields should be more relevant than the individual particle picture during $\tau \lesssim Q_s^{-1}$. Physically Q_s^2 corresponds to the transverse density of partons and is estimated by the Golec-Biernat and Wüsthoff fit, $Q_s^2 = Q_0^2(x_0/x)^\lambda A^{1/3}$, where A is the atomic number. We can expect Q_s around $1 \sim 2$ GeV for the Relativistic Heavy Ion Collider (RHIC) and $2 \sim 3$ GeV for the Large Hadron Collider (LHC) in case of $A = 197$ (Au-Au collision) assuming relevant p_t is ~ 1 GeV. This transient but still coherent gluon matter, which is often referred to as "Glasma" [7], should melt toward a quark-gluon plasma.

The physical property of Glasma has been mainly analyzed by numerical simulations in the lattice discretized formulation [4, 5, 6, 7, 8, 9]. In this paper we aim to approach Glasma in an analytical way along a similar line to the near-field expansion proposed by Fries-Kapusta-Li [10]. The analytical method is desirable for a deeper insight into the Glasma, which presumably exists up to $\tau \lesssim Q_s^{-1} \sim 0.1$ fm in the Au-Au (central) collision at RHIC energy, $\sqrt{s} = 200$ GeV/nucleon, or even longer depending on the interpretation of the Glasma. In particular, the problem of *early thermalization* still has interesting unanswered questions [11]. We will specifically address the following; is there any unstable mode growing around the initial CGC fields right after the collision? If any, it could speed up thermalization (or isotropization) even in the classical regime ($\tau \lesssim Q_s^{-1}$), besides non-Abelian plasma instabilities [12, 13, 14, 15] which take place at later times. The pioneering numerical simulation [9] suggests the existence of "Glasma instability", though the literal time scale of instability seems to be greater than Q_s^{-1} by three order of magnitude, probably because of the choice of tiny instability seeds. The delay in the instability onset has also been pointed out in the Hard Expanding Loop (HEL) approach to non-Abelian plasma instabilities [15].

We shall start with the boost-invariant CGC solution and estimate an initial energy density. It is convenient to adopt the Bjorken coordinates spanned by the proper time $\tau = \sqrt{t^2 - z^2}$ and the space-time rapidity $\eta = \frac{1}{2}\ln[(t+z)/(t-z)]$. The radial gauge $A_\tau = 0$ is understood throughout this work. The canonical momenta (chromo-electric fields) are read in this gauge as

$$E^i = \tau\partial_\tau A_i\ , \quad E^\eta = \tau^{-1}\partial_\tau A_\eta\ . \tag{1}$$

It should be mentioned that the metric is $g_{\tau\tau} = 1$, $g_{\eta\eta} = -\tau^2$, and $g_{xx} = g_{yy} = -1$ in accord with the convention in Refs. [9, 16]. The equations of motion derived from Hamilton's equations lead us to

$$\partial_\tau E^i = \tau^{-1} D_\eta F_{\eta i} + \tau D_j F_{ji}\ , \quad \partial_\tau E^\eta = \tau^{-1} D_j F_{j\eta}\ . \tag{2}$$

These are the basic equations for the classical description valid in the early stage right after the collision. The initial condition is uniquely determined by boundary matching at singularities of the color sources $\rho^{(1)}(\boldsymbol{x}_\perp)\delta(x^-)$ and $\rho^{(2)}(\boldsymbol{x}_\perp)\delta(x^+)$ representing the propagation of Lorentz contracted nuclei [2, 16], as follows;

$$\begin{aligned} &A_{i(0)} = \alpha_i^{(1)} + \alpha_i^{(2)}\ , \quad A_{\eta(0)} = 0\,, \\ &E^i_{(0)} = 0\ , \quad E^\eta_{(0)} = ig\big[\alpha_i^{(1)}, \alpha_i^{(2)}\big]\,, \end{aligned} \tag{3}$$

where $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$ are the gauge fields at $\tau < 0$ associated with the right-moving nucleus along the x^+ axis and the left-moving nucleus along the x^- axis [17]. It takes a pure-gauge form, $\alpha_i(\boldsymbol{x}_\perp) = -(1/ig)V(\boldsymbol{x}_\perp)\partial_i V^\dagger(\boldsymbol{x}_\perp)$, with the Wilson line defined by

$$V^\dagger(\boldsymbol{x}_\perp) = \mathcal{P}\exp\left[-ig\int dz^-\,\frac{1}{\boldsymbol{\partial}_\perp^2}\rho^{(1)}(\boldsymbol{x}_\perp)\delta(z^-)\right]\ , \tag{4}$$

for $\alpha_i^{(1)}$. The Wilson line for $\alpha_i^{(2)}$ is given by replacement of x^- and $\rho^{(1)}(\boldsymbol{x}_\perp)$ by x^+ and $\rho^{(2)}(\boldsymbol{x}_\perp)$ in the above expression. We can compute the expectation value of physical observables by means of the average over the random color distribution inside nuclei using

$$\big\langle \rho_a^{(m)}(\boldsymbol{x}_\perp)\rho_b^{(n)}(\boldsymbol{y}_\perp)\big\rangle = g^2\mu^2\,\delta^{mn}\,\delta_{ab}\,\delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)\ . \tag{5}$$

Here μ is the only dimensionful scale in the McLerran-Venugopalan (MV) model and related to the saturation

scale Q_s. We will later present all dimensionful quantities in unit of μ.

Let us evaluate the initial energy density of the fields (3) at $\tau = 0$ with the color source average (5). To do this, we need to take an average of four Wilson lines $\sim \langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)V(\boldsymbol{u}_\perp)V^\dagger(\boldsymbol{v}_\perp)\rangle$. We can find an algebraic technique in the appendix of Ref. [18] and it is even possible to write a formal expression down for more generic color structure [19]. Alter all, it turns out that the transverse fields are vanishing and the longitudinal chromo-magnetic fields, $B^\eta_{(0)} = F_{12(0)}$, are [8]

$$\frac{g^2}{(g^2\mu)^4}\cdot\Big\langle 2\mathrm{tr}\big(B^\eta_{(0)}\big)^2\Big\rangle = \frac{1}{32}N_c(N_c^2-1)\,\sigma^2\;. \tag{6}$$

The number of color is $N_c = 3$ in QCD. We defined σ resulting from the two-point function in terms of $\alpha_i^{(m)}$. In order to make a direct comparison to the numerical simulation transparently, we shall make use of the lattice regularization, which gives

$$\begin{aligned}\sigma &= \frac{1}{2L^2}\sum_{n_i=1-L/2a}^{L/2a}\frac{1}{2-\cos(2\pi n_1 a/L)-\cos(2\pi n_2 a/L)} \\ &\simeq \frac{1}{2\pi}\ln(cL/a)\;. \end{aligned}\tag{7}$$

Here L is the size of the system fixed by $L^2 = \pi R_A^2$, and a is the lattice spacing. We got rid of the zero-mode $n_1 = n_2 = 0$ because of global neutrality. We numerically checked that the above logarithmic form with adjusted by a constant $c \simeq 1.36$ is a quite good approximation. Some further calculations end up with the same amount of the chromo-electric field squared; $\langle 2\mathrm{tr}(E^\eta_{(0)})^2\rangle = \langle 2\mathrm{tr}(B^\eta_{(0)})^2\rangle$. As a result, we can estimate the initial energy density as

$$\frac{g^2}{(g^2\mu)^4}\cdot\varepsilon_{(0)} = \frac{3}{4}\sigma^2 \tag{8}$$

with $N_c = 3$ substituted. This a and L dependent result should be interpreted carefully, while the quantitative output somehow agrees with the latest simulation by Lappi; our estimate by Eqs. (7) and (8) yields 0.81 and 0.90 for $L/a = 500$ and 700 which are close to 0.76 and 0.88 reported in Ref. [8]. The logarithmic singularity has been found also in Refs. [8, 10]. The singularity arises from the approximations that we regarded the colliding nuclei as infinitely thin in the longitudinal direction and that the random color distribution is uncorrelated at arbitrary microscopic scale in transverse space.

Next, we will step away from singularity located at $\tau = 0$ by the near-field expansion in terms of τ, i.e., $\mathcal{O} = \mathcal{O}_{(0)} + \mathcal{O}_{(1)}\tau + \mathcal{O}_{(2)}\tau^2 + \cdots$. The first-order corrections are vanishing, and the second-order fields are

$$\begin{aligned} &A_{i(2)} = \tfrac{1}{2}E^i_{(2)} = \tfrac{1}{4}D_{j(0)}F_{ji(0)}\;, \\ &A_{\eta(2)} = \tfrac{1}{2}E^\eta_{(0)}\;, \quad E^\eta_{(2)} = \tfrac{1}{2}D_{j(0)}F_{j\eta(2)}\;, \end{aligned}\tag{9}$$

where

$$\begin{aligned} &F_{ji(0)} = -ig\big(\big[\alpha_j^{(1)},\alpha_i^{(2)}\big] + \big[\alpha_j^{(2)},\alpha_i^{(1)}\big]\big)\;, \\ &F_{j\eta(2)} = \tfrac{1}{2}D_{j(0)}E^\eta_{(0)}\;, \end{aligned}\tag{10}$$

which physically represent the initial longitudinal and second-order transverse chromo-magnetic fields.

Using these expressions we calculate the contributions to the energy density of order τ^2 to find the same amount of chromo-magnetic and chromo-electric fields again. Since the initial state has non-zero longitudinal fields, it follows that the cross terms between the zeroth and second-order terms give

$$\begin{aligned} &\frac{g^2}{(g^2\mu)^4}\cdot 2\Big\langle 2\mathrm{tr}\big(B^\eta_{(2)}B^\eta_{(0)}\big)\Big\rangle = \frac{g^2}{(g^2\mu)^4}\cdot 2\Big\langle 2\mathrm{tr}\big(E^\eta_{(2)}E^\eta_{(0)}\big)\Big\rangle \\ &= -\frac{1}{32}N_c(N_c^2-1)\,\sigma\cdot\chi + \mathcal{O}(\sigma^3)\;, \end{aligned}\tag{11}$$

where we defined

$$\chi = \frac{1}{L^2}\sum_{n_i=1-L/2a}^{L/2a} \simeq \frac{1}{a^2}\;. \tag{12}$$

We dropped terms proportional to σ^3 not containing χ because $\chi \gg \sigma$ when a is small. In the same approximation the transverse fields of order τ^4 (that is, τ^2-order in the energy density) result in

$$\begin{aligned} &\frac{g^2}{(g^2\mu)^4}\cdot\Big\langle 2\mathrm{tr}\big(B^i_{(2)}B^i_{(2)}\big)\Big\rangle = \frac{g^2}{(g^2\mu)^4}\cdot\Big\langle 2\mathrm{tr}\big(E^i_{(2)}E^i_{(2)}\big)\Big\rangle \\ &= \frac{1}{64}N_c(N_c^2-1)\,\sigma\cdot\chi + \mathcal{O}(\sigma^3)\;. \end{aligned}\tag{13}$$

After all, we get the expanded series,

$$\begin{aligned} \frac{g^2}{(g^2\mu)^4}\cdot\varepsilon &\simeq \frac{g^2}{(g^2\mu)^4}\Big[\varepsilon_{(0)} + \varepsilon_{(2)}\tau^2\Big] \\ &= \frac{1}{32}N_c(N_c^2-1)\sigma\bigg[\sigma - \pi\frac{(g^2\mu\tau)^2}{(g^2\mu a)^2}\bigg]\;. \end{aligned}\tag{14}$$

It is obvious from Eq. (14) that the τ expansion behaves badly for small value of a, which is also clear by the dotted curve in Fig. 1 that plots Eq. (14).

The naive τ expansion is, in fact, ill-defined. It is because, as pointed out in Ref. [7], the energy density behaves as $\sim(\ln\tau)^2$ near $\tau = 0$ when the colliding nuclei are infinitely thin. Therefore, the naive Taylor expansion around $\tau = 0$ is meaningless. Nevertheless, we stress that we can derive meaningful information from Eq. (14); we know that the asymptotic form $\sim(\ln\tau)^2$ in the $a \to 0$ limit and we also know that the regularized expansion $\sim c_1\ln(L/a)[\ln(L/a) + c_2(\tau/a)^2]$ with a kept finite. The simplest analytical function satisfying these two requirements is $\sim c_1\{\ln[L^2/(a^2 - c_2\tau^2)]\}^2$, that means,

$$\frac{g^2}{(g^2\mu)^4}\cdot\varepsilon \simeq \frac{3}{4}\bigg\{\frac{1}{4\pi}\ln\bigg[\frac{c^2(g^2\mu L)^2}{(g^2\mu a)^2 + \pi(g^2\mu\tau)^2}\bigg]\bigg\}^2\;. \tag{15}$$

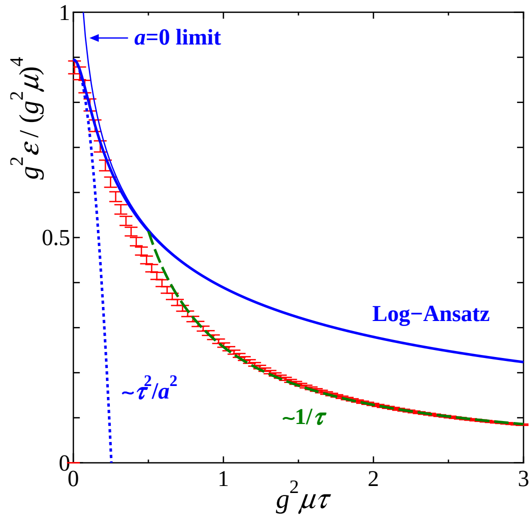

FIG. 1: Comparison of the energy density in case of $L/a = 700$; the data with error bar is taken from Ref. [8]. The dotted and solid curves represent the naive expansion in Eq. (14) and the log-ansatz in Eq. (15), respectively. The dashed curve scales as $1/\tau$ whose starting point is chosen at $g^2\mu\tau = 0.5$, meaning that the "formation time" [4] being $g^2\mu\tau_D \sim 0.5$.

The comparison to data obtained in the numerical simulation is presented in Fig. 1. This simple log-ansatz works well as long as $g^2\mu\tau \lesssim 0.5$ and is stable under the $a \to 0$ limit as shown by a thin curve in the figure.

So far, we reached an ansatz (15) with the infrared cut-off provided by the nucleus size L in a heuristic way. In reality, however, the long-ranged correlation should be cut off by the confining scale $\sim \Lambda_{\rm QCD}^{-1}$ rather than L. In the continuum limit, hence, the initial energy density in the central collision at $\tau \ll (g^2\mu)^{-1}$ should be estimated by

$$\varepsilon = \frac{3}{16\pi^2 g^2}(g^2\mu)^4\Big\{\ln(\Lambda_{\rm QCD}^{-1}/\tau)\Big\}^2 . \qquad (16)$$

In writing Eq. (16) we put the constants c and π appearing in Eq. (15) away into ambiguity of $\Lambda_{\rm QCD}$. We remark that the $\Lambda_{\rm QCD}$-dependence would be milder than the above in the regime after the "formation time" as investigated in Ref. [4].

It is interesting to apply our formula (16) to the Au-Au collision at RHIC in the physical unit. We make use of the parameter choice as commonly used in the numerical simulation, i.e., $g^2/4\pi = 1/\pi$ and $g^2\mu = 2$ GeV [4, 5, 6, 10]. As for the confining scale, we vary $\Lambda_{\rm QCD}^{-1}$ from 1 fm to 12 fm $\simeq L$. The results are summarized as follows;

$\Lambda_{\rm QCD}^{-1}$ [fm]	1	3	5	8	10	12
$\varepsilon(\tau=0.1$ fm) [GeV$\cdot$ fm^{-3}]	53	115	152	191	211	228
corrected [GeV$\cdot$ fm^{-3}]	36	77	102	128	142	153

Our log-ansatz overestimates the energy density and the third row shows the corrected values with a factor 0.67 inferred from Fig. 1. This factor might depend on $\Lambda_{\rm QCD}^{-1}$, and thus, the numbers listed in the second and third rows should be considered as the upper and lower bounds.

It is a natural choice to take the confining scale as the nucleon size ~ 1 fm, and the estimate of the initial energy density is then $\varepsilon(\tau=0.1\text{ fm}) = 40 \sim 50$ GeV$\cdot$ fm^{-3}. This value is significantly smaller than the previous estimates, 130 GeV$\cdot$ fm^{-3} in Ref. [8] and 260 GeV$\cdot$ fm^{-3} in Ref. [10], reflecting difference between the choices $\Lambda_{\rm QCD}^{-1} = 1$ fm and $\Lambda_{\rm QCD}^{-1} = L \sim 12$ fm, but rather consistent with the simulation with color neutrality of finite nuclei taken into account [5] that found $\epsilon(\tau=\tau_D\simeq 0.3\text{ fm}) = 7.1 \sim 40$ GeV$\cdot$ fm^{-3}.

When $g^2\mu\tau$ becomes larger, the energy density comes to scale as $\sim \tau_0/\tau$ because of (almost free streaming) longitudinal expansion [9, 11]. It should be noted that the scaling law in the classical regime is different from the (one-dimensional) hydrodynamic one $\sim (\tau_0/\tau)^{4/3}$. For reference we plot the scaling behavior $\varepsilon(\tau)/\varepsilon(\tau_0) = \tau_0/\tau$ in Fig. 1 indicated by the dashed curve with a choice of $g^2\mu\tau_0 = 0.5$. The expanding system at late times is dilute so that this scaling behavior is to be justified by the solution of Eq. (2) in the weak field limit, which in fact scales as [2, 8]

$$A_i \sim A_\eta \sim 1/\sqrt{\tau}\,. \qquad (17)$$

The energy density is dominated only by the Abelian part $\sim (\partial A)^2$, hence it follows the τ_0/τ scaling. We would comment on a curious observation that, if we extrapolate our estimate $\varepsilon(\tau=0.1\text{ fm})$ up to $\tau = 1$ fm assuming the τ_0/τ scaling, the initial energy density obtained accordingly is very close to the standard estimate by means of the Bjorken formula; $\epsilon(\tau=1\text{ fm}) \sim 5.1$ GeV$\cdot$ fm^{-3}.

We shall next consider the problem of instability in the rest of this paper. We treat fluctuations δA_i and δE^i in the linear order around the boost-invariant CGC background which we discussed above. As formulated in Ref. [16], δE^η should be constrained by the Gauss law and we drop δA_η because it is accompanied by τ^2 from the metric. In what follows we regard η-dependent fluctuations, δA_i and δE^i, as the "soft" fields and η-independent CGC fields as the "hard" background which brings about instability. The linearized equations of motion are

$$\begin{aligned}\tau\partial_\tau\delta\tilde{A}_i &= \delta\tilde{E}^i\,,\\ \partial_\tau\delta\tilde{E}^i &= -\tau^{-1}\nu^2\delta\tilde{A}_i + \tau\,G_{ij}^{-1}\delta\tilde{A}_j\,,\end{aligned} \qquad (18)$$

where we introduced the Fourier transform $\delta\tilde{A}_i$ from η to the wave number ν (i.e. $\partial_\eta^2\delta A_i(\eta) \to -\nu^2\delta\tilde{A}_i(\nu)$) and we denoted the inverse of the transverse background gluon propagator as G_{ij}^{-1} whose definition is

$$G_{ij}^{-1ab} = \delta_{ij}(D_kD_k)^{ab} - (D_iD_j)^{ab} + 2gf^{acb}F_{ij}^c\,. \qquad (19)$$

We note that, in correspondence to the HEL approach, the color current encoding the anisotropic distribution

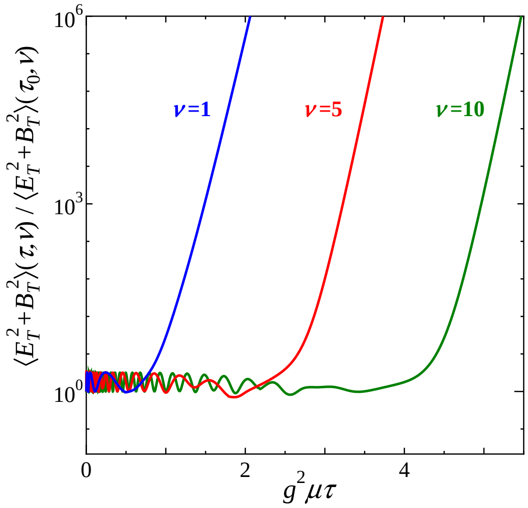

FIG. 2: Instability tendency with the initial CGC background fixed at $\tau = 0$ in case of $L/a = 700$ for different wave numbers $\nu = 1$, 5, and 10 from the left to the right.

of hard background is identified as $j_i^a = \left[G_{ij}^{-1ab} - \delta^{ab}\left(\delta_{ij}\partial_k\partial_k - \partial_i\partial_j\right)\right]\delta\tilde{A}_j^b$. Although it is not clear whether this current could have anything to do with that in the HEL approach after the ensemble average, we can shortly confirm that instability may occur even in the purely classical regime.

It is easy to solve Eq. (18) to obtain $\delta\tilde{A}_i$ as a function of a given constant CGC background, $G_{(0)}^{-1}$ at initial time, because we can diagonalize $G_{(0)}^{-1}$ in a proper basis of $\delta\tilde{A}_i$. If we write its eigenvalues as λ, the solution is

$$\delta\tilde{A}_i(\lambda) = c_{1i}\,\mathrm{Re}I_{i\nu}(\sqrt{\lambda}\tau) + c_{2i}\,\mathrm{Im}I_{i\nu}(\sqrt{\lambda}\tau)\,, \tag{20}$$

for $\lambda > 0$ (which exponentially grows) and

$$\delta\tilde{A}_i(\lambda) = c_{1i}\,\mathrm{Re}J_{i\nu}(\sqrt{|\lambda|}\tau) + c_{2i}\,\mathrm{Im}J_{i\nu}(\sqrt{|\lambda|}\tau)\,, \tag{21}$$

for $\lambda < 0$ (which oscillatorily diminishes), where $J_n(x)$ and $I_n(x)$ are the first-kind and modified Bessel functions. These special functions are singular as $(\sqrt{|\lambda|}\tau)^{\pm i\nu}$ which furiously rotates in complex space as $\tau \to 0$. At later time when the asymptotic behavior (17) realizes due to expansion, λ is no longer a constant but a function of time like $\sim \xi/\tau$ with some dimensionful constant ξ, which results in the solutions $I_{2\pm i\nu}(2\sqrt{\xi\tau})$ for $\xi > 0$ and $J_{2\pm i\nu}(2\sqrt{|\xi|\tau})$ for $\xi < 0$. For other general cases, the τ-dependence in the eigenvalue is intricate, and one needs to solve Eq. (18) numerically.

We do not treat such a general case but what we will pursue here is to clarify whether the *initial* CGC fields could induce exponential growth for soft degrees of freedom. We consider this because the initial CGC configurations at $\tau = 0$ are the most (spatially) anisotropic (namely, large longitudinal and zero transverse fields) and thus anticipated to cause the most unstable modes. Under such extreme circumstances we expect that the physics of instability becomes clear. Thus, as a first trial, it should be an appropriate starting point. We can say that what we will do is to extract the *tendency* toward instability under the presence of the CGC background. For simplicity we will focus on the case that fluctuations are uniform in transverse space, i.e. $\partial_i\delta\tilde{A} = 0$, which should be the most unstable, as adopted in Ref. [15].

We should calculate the Gaussian average (5) of the transverse chromo-magnetic field $\langle (B_i)^2\rangle \simeq \langle (\nu\delta\tilde{A}_i)^2\rangle$ and the chromo-electric field $\langle (E_i)^2\rangle \simeq \langle (\tau\partial_\tau\delta\tilde{A}_i)^2\rangle$ which were as small as $\sim \tau^4$ previously but non-zero this time with fluctuations depending on η, contributing to the longitudinal pressure [9]. The straightforward calculation is, however, technically hard. We will approximately do it by picking the mean value up,

$$\left\langle G_{(0)ij}^{-1ab}\right\rangle = \delta_{ij}\delta^{ab}\bar{\lambda} = -\delta_{ij}\delta^{ab}\frac{3}{8}\sigma\,(g^2\mu)^2\,, \tag{22}$$

where σ is defined in Eq. (7), and taking the ensemble average over its dispersion,

$$\left\langle G_{(0)ik}^{-1ac}G_{(0)kj}^{-1cb}\right\rangle - \delta_{ij}\delta^{ab}\bar{\lambda}^2 = \delta_{ij}\delta^{ab}\delta\lambda^2 = \delta_{ij}\delta^{ab}\frac{3}{8}\chi\,(g^2\mu)^2, \tag{23}$$

where χ is defined in Eq. (12).

Because $\bar{\lambda}$ is negative, the soft fluctuations in the vicinity of the averaged CGC background are stable belonging to the type of solution (21). The eigenvalue of $G_{(0)}^{-1}$ distributes according to random CGC configurations and spreads from $\bar{\lambda}$ with the dispersion $\delta\lambda$, meaning that some CGC configurations may have negative λ. That is, if we evaluate,

$$\left\langle \mathcal{O}[\delta\tilde{A}(\lambda)]\right\rangle \simeq \int_{-\infty}^{\infty} d\lambda\,\mathcal{O}[\delta\tilde{A}(\lambda)]\,e^{-(\lambda-\bar{\lambda})^2/2\delta\lambda^2}\,, \tag{24}$$

using Eq. (20), the contributions near $\lambda \simeq \bar{\lambda}$ dominate only when time is small until the negative λ constituents grow up as time elapses. The transverse field strengths obtained in this way are plotted in Fig. 2.

To draw Fig. 2, we chose the initial time $g^2\mu\tau_0 = 0.001$ at which we set c_1 and c_2 of Eq. (20) or (21) by the initial condition, $\delta\tilde{A}_i = c/\sqrt{\nu}$ and $\delta\tilde{E}_i = \tau_0\partial_\tau\delta\tilde{A}_i = c\sqrt{\nu}$, inspired by quantum fluctuations discussed in Ref. [16]. It is interesting to see that this specific initial condition ($\delta\tilde{E}_i \sim \nu\delta\tilde{A}_i$) makes $\langle (B_i)^2\rangle \simeq \langle (\nu\delta\tilde{A}_i)^2\rangle$ and $\langle (E_i)^2\rangle$ comparable to each other, leading to their almost alternate oscillations. That is why the sum of transverse field strengths depicted in Fig. 2 never come close to zero, which makes a contrast to the results in Ref. [15].

We can conclude that there is certainly the tendency toward instability associated with initial CGC background. The onset of instability in the present case is located much earlier than preceding works. It is because we only investigated the strongest instability encompassed in the initial CGC fields. Because the CGC background itself evolves with time, in fact, the genuine growth of instability should be slower and weaker than

shown in Fig. 2. Nevertheless, we can learn the qualitative character of the "Glasma" instability. The intuitive picture is as follows. The soft fluctuations of gluon fields are non-Abelian charged and feel a force under influence from the CGC background. The ensemble of random CGC distribution contains not only color fields which suppress the color current provided by charged soft fluctuations but also color fields which amplify the current. Although the current is suppressed on average, the large τ behavior is predominantly determined by mixture of CGC fields which enhance the input current. Therefore, we think that it is rare fluctuation in the CGC ensemble from which the Glasma instability can occur.

It is necessary to deal with $\lambda(\tau)$ as not a constant but a function of τ in order to quantify instability further. In our treatment mentioned above we dropped the effect of longitudinal expansion for the hard part, while the exponential growth should take a form of $I_{2\pm i\nu}(2\sqrt{\xi\tau}) \sim \tau^{-1/2}\exp[2\sqrt{\xi\tau}]$ asymptotically when $\lambda \sim \xi/\tau$, as we remarked before. The analytical estimate of ξ deserves future clarification. Also, we have to evaluate $\bar{\lambda}$ and $\delta\lambda$ in a resummed form like Eq. (15) beyond the naive expressions (22) and (23). As a matter of fact, the growth rate seems to be determined by $\bar{\lambda}$ and $\delta\lambda$ regardless of ν in view of our results in Fig. 2. Quantitative details of an analytical description should be improved with guided by systematic instability studies in the numerical simulation in the future.

In summary, we developed an analytic formula to estimate the initial energy density. Our conclusion is $\epsilon(\tau = 0.1\ \text{fm}) = 40 \sim 50\ \text{GeV}\cdot\text{fm}^{-3}$ in the (central) Au-Au collision at RHIC. The uncertainty comes from the infrared cut-off (or confining) scale. Also, we analyzed the tendency toward instability in the presence of the initial CGC background fixed at $\tau = 0$. We found that there exist unstable modes as a result of the ensemble average of random CGC configurations, some of which strengthen the color current brought in by soft fluctuations. Although the Glasma instability might have a connection to non-Abelian plasma instabilities at a deeper level, we would emphasize that we could understand the Glasma instability not relying on the picture of plasma instabilities that are premised on anisotropic momentum distribution of hard particles. The bottom line is, thus, that the Glasma instability exists from $\tau = 0$ even when the particle picture is irrelevant yet.

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